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On the Onsager problem for Potts models

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Abstract. It is shown how the calculation of partition functions for all Potts models might be reduced to the calculation of $\mathrm{Tr}(\gamma i_1 \ldots \gamma i_{pn})$ where the γ stand for generators of a generalised Clifford algebra. Then the expression for $\mathrm{Tr}(\gamma i_1 \ldots \gamma i_s)$ for an arbitrary collection of such γ matrices is derived.

1. Introduction

The transfer matrix technique for a statistical system with the most general translational invariant and globally symmetric Hamiltonian on a two-dimensional lattice leads to appropriate algebras of operators which are algebra extensions of the type naturally associated with the lattice-grading groups.

If the symmetry group of a Hamiltonian is the Z_n cyclic group then the algebra generated by the transfer matrix approach is, in most of the known cases, the generalised Clifford algebra $C_k^{(n)}$ [1]. It is due to its properties that the given model has the duality property [2-4].

In this paper we investigate the consequences of the simple fact that transfer matrices for all Potts models on the torus are just specific elements of the $C_{2p}^{(n)}$ algebra. Before we do this we give in § 2 some preliminary remarks about $C_{2p}^{(n)}$ algebras [1] as well as some other necessary generalisations [2].

The motivation for the investigation presented comes from Baxter [12] who writes: 'The only hope that occurs to me is just as Onsager (1944) and Kaufmann (1949) originally solved the zero-field Ising model by using the algebra of spinor operators, so there may be similar algebraic methods for solving the eight-vertex and Potts models'.

2. Preliminaries

In this section we introduce appropriate generalisations of γ matrices and cosh functions, to be used later on while investigating the algebraic structure of transfer matrices for Potts models.

We start with the $C_{2p}^{(n)}$ generalised Clifford algebra [1] which is defined as an associative algebra over complex numbers $\mathbb{C}$, generated by $\gamma_1, \ldots, \gamma_{2p}$ matrices satisfying

$$\omega\gamma_i\gamma_j = \gamma_j\gamma_i \qquad i<j; \qquad \gamma_i^n = \mathbb{1}; \qquad i,j=1,\ldots,2p$$
$$\omega = \exp(2\pi \mathrm{i}/n). \tag{2.1}$$

The algebra of $(n^P \times n^P)$ matrices has—up to equivalence—only one irreducible and faithful representation. Its generators $\{\gamma_i\}_1^{2p}$ can be represented as tensor products of generalised $(n \times n)$ Pauli matrices $\sigma_1, \sigma_2, \sigma_3$ in complete analogy with the usual Clifford algebra case. One should however distinguish the case of n being odd from that of n being even [1], namely for n odd:

$$\sigma_1 = (\delta_{i+1,j}), \qquad \sigma_2 = (\delta_{i+1,j}\omega^j)$$
$$\sigma_3 = \sigma_1^{n-1}\sigma_2 = (\omega^i\delta_{ij}) \tag{2.2}$$

where $i, j \in Z'_n = \{0, 1, \ldots, n-1\}$, i.e.

$$\sigma_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 1 & 0 & \ldots & 0 \\ & & & \ldots & & \\ 0 & 0 & 0 & 0 & \ldots & 1 \\ 1 & 0 & 0 & 0 & \ldots & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & \omega & 0 & 0 & \ldots & 0 \\ 0 & 0 & \omega^2 & 0 & \ldots & 0 \\ & & & \ldots & & \\ 0 & 0 & 0 & 0 & \ldots & \omega^{n-1} \\ 1 & 0 & 0 & 0 & \ldots & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & & & \\ & \omega & & \\ & & \omega^2 & \\ & & & \omega^{n-1} \end{pmatrix}.$$

For n even σ_2 and σ_3 are different, i.e.

$$\sigma_2 = (\xi^{2i+1}\delta_{i+1,j}), \qquad \sigma_3 = \xi\sigma_1^{n-1}\sigma_2 \tag{2.3}$$

where ξ is the *primitive* $2n$th root of unity.

With the use of generalised Pauli matrices one readily finds a Kronecker product representation of γ. We choose the following one:

$$\begin{aligned}
\gamma_1 &= \sigma_3 \otimes I \otimes I \otimes \ldots \otimes I \otimes I \\
\gamma_2 &= \sigma_1 \otimes \sigma_3 \otimes I \otimes \ldots \otimes I \otimes I \\
&\vdots \\
\gamma_p &= \sigma_1 \otimes \sigma_1 \otimes \ldots \otimes \sigma_1 \otimes \sigma_3 \\
\gamma_{p+1} &\equiv \bar{\gamma}_1 = \sigma_2 \otimes I \otimes I \otimes \ldots \otimes I \otimes I \\
\gamma_{p+2} &\equiv \bar{\gamma}_2 = \sigma_1 \otimes \sigma_2 \otimes I \otimes I \\
&\vdots \\
\gamma_{2p} &\equiv \bar{\gamma}_p = \sigma_1 \otimes \sigma_1 \otimes \ldots \otimes \sigma_1 \otimes \sigma_2.
\end{aligned} \tag{2.4}$$

The basic importance of $C_{2p}^{(n)}$ algebra for us relies on the observation that transfer matrices for both planar and standard Potts models are just specific polynomials in the above γ matrices.

Apart from the generalisation of the Clifford algebra we shall also need 'generalised cosh' functions and appropriate projection matrices [2].

We start with the 'cosh' function. Let x be any element of an associative finite-dimensional algebra (for example, a number or matrix). We define

$$f_i(x) = \sum_{k=0}^{\infty} \frac{x^{nk+i}}{(nk+i)!}, \qquad i \in Z'_n. \tag{2.5}$$

Then

$$\sum_{i=0}^{n-1} f_i(x) = e^x, \qquad f_i(\omega x) = \omega^i f(x), \qquad i \in Z'_n. \tag{2.6}$$

From (2.6) one easily derives

$$f_i(x) = \frac{1}{n} \sum_{k=0}^{n-1} \omega^{-ki} \exp(\omega^k x), \qquad i \in Z'_n. \tag{2.7}$$

Most of the identities satisfied by cosh and sinh functions generalise straightforwardly to the case of f_i; for example

$$\sum_{i=0}^{n-1} f_i(x) f_{k-i}(x) = f_k(2x), \qquad k \in Z'_n. \tag{2.8}$$

These f_i functions are shown to be important also while considering the duality property of the planar Potts models [3] because the eigenvalues χ_k; $k \in Z'_n$ (with $\chi_k = \chi_{-k}$) of the interaction matrix are expressed by f_i according to the formula [2]

$$n \sum_{i=0}^{n-1} f_i(a) f_{i-k}(a) = \chi_k(a) \tag{2.9}$$

where a is a parameter.

Finally we introduce adequate projection operators. Let U be a matrix (or number or just an element of an associative algebra) satisfying $U^n = \mathbb{1}$. We define

$$V = \frac{1}{n} \sum_{i=0}^{n-1} U^i \tag{2.10}$$

then

$$V^2 = V \tag{2.11}$$

as could easily be shown.

If one now introduces a set $\{V_k\}_0^{n-1}$ of projection operators

$$V_k = \frac{1}{n} \sum_{i=0}^{n-1} \omega^{-ki} U^i, \qquad k \in Z'_n, \tag{2.12}$$

one finds that

$$V_k V_l = \delta_{kl} V_k. \tag{2.13}$$

This ends the preliminary section. In the following we shall investigate the structure of the transfer matrix, mostly for the planar Potts model, using the following standard operators:

$$\chi_k = I \otimes \ldots \otimes I \otimes \sigma_1 \otimes I \otimes \ldots \otimes I \qquad p \text{ terms} \tag{2.14}$$

$$Z_k = I \otimes \ldots \otimes I \sigma_3 \otimes I \otimes \ldots \otimes I \qquad p \text{ terms} \tag{2.15}$$

where σ_1 and σ_3 (together with I: $(n \times n)$ matrices) are situated on the kth site from the left-hand side.

3. The structure of transfer matrices

The transfer matrix approach has led the authors of [3-5] to the use of $C_{2p}^{(n)}$ algebras, although this observation does not seem to be realised by the authors mentioned.

Meanwhile it is a very important fact that the transfer matrix M is just an element of $C_{2p}^{(n)}$; hence it is a specific polynomial in γ matrices (2.4). Due to this, the problem of determining the partition function might be reduced to the problem of calculating traces: $\mathrm{Tr}\, \gamma_{i_1} \ldots \gamma_{i_k}$ (modulo eventual combinatorial complexity). This point of view is known to lead to the exact solution of the Onsager problem, with the use of Pfaffians at the final stage [6] of computations, for the Ising model.

We are now going to investigate algebraic properties of transfer matrices for Potts models. Let us assign to the torus $p \times q$ lattice (p rows, q columns) a set of states

$$\mathscr{X} = \{(s_{ik});\ s_{ik} \in Z_n\}$$

where the multiplicative realisation of the Z_n cyclic group is chosen and s_{ik}, ($s_{ik} \in \{\omega^r\}_0^{n-1}$), denotes a matrix element of the $p \times q$ matrix.

The total energy for the standard Potts model is then given by

$$-\frac{E(s_{ik})}{kT} = \sum_{i,k}^{p,q} [a\delta(s_{ik} - s_{i,k+1}) + \delta(s_{ik} - s_{i+1,k})] \tag{3.1}$$

while for the planar Potts model

$$-\frac{E(s_{ik})}{kT} = \sum_{i,k}^{p,q} [a(\bar{s}_{ik}s_{i,k+1} + \bar{s}_{i,k+1}s_{ik}) + b(\bar{s}_{ik}s_{i+1,k} + \bar{s}_{i+1,k}s_{ik})]. \tag{3.2}$$

The transfer matrix M is represented as a product

$$M = BA \tag{3.3}$$

where in the case of the standard Potts model [4, 7]

$$B = \prod_{k=1}^{p} \exp\left(\frac{1}{n} \sum_{i=0}^{m-1} (Z_k^+ Z_{k+1})^i\right) \tag{3.4}$$

and

$$A = \prod_{k=1}^{p} \left(\mathbb{1}\, \mathrm{e}^a + \sum_{i=1}^{m-1} X_k^i\right). \tag{3.5}$$

The corresponding expressions for A and B ($n^p \times n^p$) matrices in the case of the planar Potts model [4, 7] are given by

$$B = \prod_{k=1}^{p} \exp(Z_k^+ Z_{k+1} + Z_{k+1}^+ Z_k) \tag{3.6}$$

and

$$A = \prod_{k=1}^{p} \left(\sum_{i=0}^{n-1} \lambda_i X_k^i\right) \tag{3.7}$$

where

$$\lambda_i = \exp(2a\ \mathrm{Re}\ \omega^i). \tag{3.8}$$

The boundary conditions corresponding to the torus lattice result in the requirement

$$Z_{p+1} = Z_1. \tag{3.9}$$

The interaction $(n \times n)$ matrices $\hat{a}$ for corresponding models are given by

(planar)

$$\hat{a}(a) = \sum_{i=0}^{n-1} \lambda_i \sigma_1^i = \begin{pmatrix} \lambda_0 & \lambda_1 & \lambda_2 & \dots & \lambda_{n-1} \\ \lambda_{n-1} & \lambda_0 & \lambda_1 & \dots & \lambda_{n-2} \\ & & & \dots & \\ \lambda_1 & \lambda_2 & \lambda_3 & \dots & \lambda_0 \end{pmatrix} \tag{3.10}$$

(standard)

$$\hat{a}(a) = \mathbb{1}\, e^a + \sum_{i=1}^{n-1} \sigma_1^i = \begin{pmatrix} e^a & 1 & 1 & \dots & 1 & 1 \\ 1 & e^a & 1 & \dots & 1 & 1 \\ & & & \dots & & \\ 1 & 1 & 1 & \dots & 1 & e^a \end{pmatrix}. \tag{3.11}$$

A knowledge of interaction matrices enables one to represent the matrix A in an exponential form after a dual parameter a^* has been introduced. As (3.11) is a special case of (3.10) we shall proceed to do so only for the planar Potts model.

If one defines the dual parameter a^* [2, 8] according to

$$\det \hat{a}(a^*) = n^n [\det \hat{a}(a)]^{-1} \tag{3.12}$$

then the matrix A can be written in the form

$$A = [\det \hat{a}(a)]^{p/n} \exp\left(a^* \sum_{k=1}^{p} (X_k + X_k^+) \right). \tag{3.13}$$

It should be noted that the factor in front of the exponential is known as

$$\det \hat{a}(a) = \prod_{k=0}^{n-1} \chi_k(a), \tag{3.14}$$

where χ_k are given by (2.9).

Due to the property $X_k^n = Z_k^n = \mathbb{1}$ both A and B operators could be expressed as simple polynomials in X and Z with the coefficients just being products of $f_i(a^*)$ and $f_j(b)$. The boundary conditions, as in the Ising model case, give rise to projection operators (n of them). First we shall analyse the B matrix of the planar Potts model with boundary conditions being taken into account. In order to do that we extract from B the boundary term and notice (for n odd) that [7]

$$\exp(bZ_p^+ Z_1) = \exp(bU\bar{\gamma}_p^{-1}\gamma_1) \tag{3.15}$$

where

$$U = \prod_{k=1}^{p} \gamma_k^{-1}\bar{\gamma}_k = \omega^{-1} \otimes^p \sigma_1, \tag{3.16}$$

(hence $U^n = \mathbb{1}$).

The n-even case differs only in a factor [7], for example

$$Z_p^+ Z_1 = \xi^{-1} U \bar{\gamma}_p^{-1} \gamma_1 .$$

Therefore from now on we shall write formulae only for *n odd.*

The (3.15) term and its Hermitian conjugate may be expressed in terms of the projection matrices V_k defined by (2.12) and, if in addition a set of $\{B_k\}_0^{n-1}$ matrices is introduced according to

$$B_k = \exp\left(b \sum_{\alpha=1}^{p-1} \bar{\gamma}_\alpha^{-1} \gamma_{\alpha+1} \right) \exp(b\omega^k \bar{\gamma}_p^{-1} \gamma_1) \tag{3.17}$$

then, due to (2.13), the final expression for the B matrix reads as follows:

$$B=\sum_{k=0}^{n-1} B_k B_k^+ V_k. \tag{3.18}$$

It is now obvious that a similar structure for B can be obtained for the standard Potts model, and that for both cases the transfer matrix M is a polynomial in γ.

Because of (3.16) all V commute with the A matrix; therefore we obtain for the partition function $\mathscr{Z}$ the following formula:

$$\mathscr{Z}=\mathrm{Tr}\, M^q=\mathrm{Tr}\left(\sum_{k=0}^{n-1}[B_k B_k^+ A]^q V_k\right). \tag{3.19}$$

Already from formula (3.19) one may draw an important conclusion, namely the partition function $\mathscr{Z}$ for a finite torus lattice with Z_n symmetry is proportional to a polynomial in $f_i(a^*)$ and $f_j(b)$, the coefficients of the corresponding monomials being ω^k for some $k \in Z'_n$. This is easily seen from the fact that $X_k=\omega^{-1}\gamma_k^{-1}\bar{\gamma}_k$ (for n odd), $\bar{\gamma}_k^n=\gamma_k^n=\mathbb{1}$ and (as we shall see) because the normalised trace takes the values

$$\mathrm{Tr}(\gamma_{i_1}\dots\gamma_{i_s}) \in \{0, \omega^k;\ k=0,1,\dots,n-1\}.$$

For $n=2$ ($f_0 \equiv \cosh$, $f_1 \equiv \sinh$) this polynomial is known [6] due to the properties of the Pfaffian. For arbitrary n the form of this polynomial can also be derived [7]. However no *transparent* final formula is known to us and an adequate generalisation of the trace formula $\mathrm{Tr}(\gamma_{i_1}\dots\gamma_{i_s})$ though also already known—used together with the expression for the polynomial in γ—gives a rather complicated outcome. We hope however to achieve meaningful progress in that direction soon.

The use of the generalised Pfaffian-like formula is of course not the only way to proceed with the expression (3.19). One may also try to follow, by analogy with $n=2$, those approaches which use Grassmann algebras associated with Clifford algebras via Witt decomposition ('Fermi operators') as for example in [9] or (another method) in [10]. The appropriate generalised Grassmann algebras associated with the $C_{2p}^{(n)}$ ones ('paraFermi operators') are known [11]. Meanwhile, we conclude our temporal investigation by supplying, in the forthcoming section, a trace formula (4.1) for the arbitrary monomial in generalised γ.

4. The trace formula

Let us adopt the convention: $\mathrm{Tr}\,\mathbb{1}=1$. In the following the explicit formula for the trace of any element of the $C_{2p}^{(n)}$ algebra is derived. This also solves the problem of traces for the $C_{2p+1}^{(n)}$ algebra as $C_{2p+1}^{(n)}$ is a direct sum of n copies of $C_{2p}^{(n)}$.

The derivation has the form of a sequence of five lemmas, where (stated once for all five) $i_1,\dots,i_k,\dots,i_{kn}=1,\dots,2p$.

Lemma 1. Let $k \neq 0 \bmod n$; $k \in \mathbb{N}$. Then $\mathrm{Tr}(\gamma_{i_1}\dots\gamma_{i_k})=0$.

Proof. The same as for usual Clifford algebras. Use the U matrix defined by (3.16).

From now on S_r denotes a group of permutations of the r-elemental set. With this in mind we have the following lemma.

Lemma 2. $\mathrm{Tr}(\gamma_{i_1} \ldots \gamma_{i_{kn}}) \neq 0$ iff there exists $\sigma \in S_{kn}$ such that

(a) $i_{\sigma(1)} = i_{\sigma(2)} = \ldots = i_{\sigma(n)}$,

$i_{\sigma(n+1)} = \ldots = i_{\sigma(2n)}, \ldots, i_{\sigma(kn-n+1)} = \ldots = i_{\sigma(kn)}$.

Proof. The proof follows from the observation that, due to (2.1), if no n-tuple of the same γ exists then $\mathrm{Tr}(\ldots) = 0$. Other $k-1$ steps of the proof are reduced to this first one.

It is then trivial to note but important to realise the following lemma.

Lemma 3. $\mathrm{Tr}(\gamma_{i_1} \ldots \gamma_{i_k}) \in \{0, \omega^s; s = 0, 1, \ldots, n-1\}$.

The major problem now is to determine this value '0' or 'ω^s' for an arbitrary set of indices $i_1, \ldots, i_k$. For $n = 2$ it is the signum function that takes care of the $(-1)^s$ value of $\mathrm{Tr}(\ldots) \neq 0$. We shall therefore introduce a generalisation of the signum function according to the following definition.

Definition. The signum-like function K is a surjective map $K: S_p \to Z_n$ defined by

$$\theta_{\sigma(1)}\theta_{\sigma(2)} \ldots \theta_{\sigma(p)} = K(\sigma)\theta_1\theta_2 \ldots \theta_p$$

where

$$\omega\theta_i\theta_j = \theta_j\theta_i \qquad i<j, \qquad \theta_i^2 = 0, \qquad i, j = 1, \ldots, p.$$

For $n = 2$ these θ matrices become anticommuting matrices, i.e. the generators of Grassmann algebra, while K becomes (only for $n = 2$) the epimorphism.

Now consider a set of $\gamma_{i_1}, \ldots, \gamma_{i_{kn}}$ matrices which consists of k different n-tuples of correspondingly the same γ mixed together. Then of course there exists $\sigma \in S_{kn}$ satisfying (a) from lemma 2. In fact there are many. If one however chooses one such that

(b) $\sigma(1) < \sigma(2) < \ldots < \sigma(n)$,

$\sigma(n+1) < \ldots < \sigma(2n), \ldots, \sigma(kn-n+1) < \ldots < \sigma(kn)$

then one has the following lemma.

Lemma 4. Let $\gamma_{i_1}, \ldots, \gamma_{i_{kn}}$ be such a collection of k different n-tuples of generalised γ matrices that conditions (a) and (b) are satisfied; then

$$\mathrm{Tr}(\gamma_{i_1} \ldots \gamma_{i_{kn}}) = K(\sigma^{-1}).$$

Proof. This follows directly from the definition of the K signum-like function.

The generalisation of lemma 4 to the arbitrary case of some of the n-tuples being equal is straightforward. Bearing this in mind and from the other lemmas we have another lemma.

Lemma 5

$$\mathrm{Tr}(\gamma_{i_1} \ldots \gamma_{i_{kn}}) = \sum_{\sigma \in S_{kn}}{}' K(\sigma^{-1})\delta(i_{\sigma(1)}, \ldots, i_{\sigma(n)})$$
$$\times \delta(i_{\sigma(n+1)}, \ldots, i_{\sigma(2n)}) \times \ldots \times \delta(i_{\sigma(kn-n+1)}, \ldots, i_{\sigma(kn)}) \tag{4.1}$$

for an arbitrary collection of indices $i_1, \ldots, i_{kn}$, where δ denotes the multi-index Kronecker delta, i.e. it assigns zero to its arguments unless all of them are equal and in this case $\delta(\ldots) = 1$. The sum Σ is meant to take into account *only* these permutations $\in S_{kn}$ that satisfy the following conditions:

(b) $\sigma(1) < \sigma(2) < \ldots < \sigma(n), \sigma(n+1) < \ldots < \sigma(2n), \ldots, \delta(kn-n+1) < \ldots < \sigma(kn)$

and

(c) $\sigma(1) < \sigma(n+1) < \ldots < \sigma(kn-n+1)$.

The '(c)' condition is necessary to avoid an overcounting of σ satisfying (a).

Lemma 5 provides us then with the straightforward generalisation of the Pfaffian also for linear combinations of generalised γ as can be seen from the following lemma.

Lemma 6. Let $\hat{P} = \Sigma_{i=1}^{2r} p_i \gamma_j$ where $\{\gamma_j\}_1^{2r}$ form the set of generators for $C_{2r}^{(n)}$, while $p_i \in \mathbb{C}$; $i = 1, \ldots, 2p$. Let

$$P = \begin{pmatrix} p_1 \\ \vdots \\ p_{2r} \end{pmatrix}$$

and denote by $(P_1, P_2, \ldots, P_n) = \Sigma_{i=1}^{2r} p_{1i} p_{2i} \ldots p_{ni}$ an n-linear 'scalar product' of $P_1, \ldots, P_n$ vectors [11]. Then

$$\operatorname{Tr}(\hat{P}_1 \hat{P}_2 \ldots \hat{P}_{kn}) = \sum' k(\sigma^{-1})(P_{\sigma(1)}, P_{\sigma(2)}, \ldots, P_{\sigma(n)}) \times (P_{\sigma(n+1)}, \ldots, P_{\sigma(n)}) \ldots (P_{\sigma(kn-n+1)}, \ldots, P_{\sigma(kn)}). \tag{4.2}$$

Again Σ' means that conditions (b) and (c) are satisfied. Naturally formula (4.2) solves the problem of taking the trace from a product of *any* number of $\hat{P}$ as this trace is zero, for the number of P different from kn, $k \in \mathbb{N}$.

The formula (4.2) might be useful for our purposes if one could express matrices A and B as a product of $\hat{P}$ which however seems to be possible only for $n = 2$.

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